Chapter Fifteen

Conclusions and Reflections

With many thanks to
NRICH, 'Peter'
and all other AskNRICHers

Doing Mathematics in Different Places: an Exploration of Young People's Activities as they make Independent Use of a Web-Based Discussion Board

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Each chapter has been edited to enable it, as far as is feasible, to 'standalone'.

The chapter numbers and numbering of sub-headings has been left unchanged from the original Thesis.

However, each edited chapter has its own page numbering and any cross-references *within* the chapters and *between* chapters on the NRICH website use these (new) page numbers followed by specifying the page number(s) in the original Thesis chapters.

Where appropriate, references may be given to other chapters (not included on the website) within the full Thesis, either by specifying the Section or providing the Thesis page number(s).

If in a chapter reference is made to any appendices, then the relevant appendix is attached at the end of that chapter.

Each chapter has its own list of references.

[The Thesis title, abstract and acknowledgement pages together with a table of contents for these edited chapters and glossary from the Thesis are also included. The table of contents of the full Thesis appears after Chapter Fifteen].

Dr Libby Jared

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Chapter Fifteen

Conclusions and Reflections

15.1 Introduction

[The chapter presented here is formed from parts extracted from the final chapter of the Thesis, titled Conclusions and Reflections. Only Section 15.3 Thesis Claims and Contribution to Knowledge, and parts of Sections 15.4 Limitations to the Study, and 15.5 Research Implications are included]

This thesis claims a contribution to knowledge in five parts summarised as follows:

Claim 1: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board

Claim 2: Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy

Claim 3: The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits

Claim 4: AskNRICH can be successfully characterised using a concept of 'place', based on a modification of Gee's model of an Affinity Space, through the introduction and definition of two new concepts, Pupil Learning Place [PLP] and Second Learning Place [SLP]

Claim 5: The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools

15.3 Thesis Claims and Contribution to Knowledge

Of the five claims this thesis makes, the first is methodological in nature, the remaining four result from the process leading to the characterisation of AskNRICH as an SLP.

Claim 1: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board

The work undertaken highlighted the differences between AskNRICH and other web-boards, both in the unique nature of AskNRICH and the size of datasets studied. Furthermore, the approach needed to address the jumbled, interwoven nature of the exchanges, an aspect not specific to AskNRICH but still adding to the complexity of the task of analysis. The novel research approach devised, and reported on at length [see Thesis Chapter Six], adds to the other approaches and methods currently available. The advances include:

i. employing the combination of multiple Perspectives to allow systematic and comprehensive coverage thus producing a more wholistic view
ii. the construction of interpretative commentaries on threads
iii. the separation of mathematical content from actions in the threads and their encapsulation into two separate commentaries [exemplified in Chapters Nine and Eleven]
iv. the derivation of a set of response types used to classify posts and fragments of posts
v. a diagrammatic representation of the linkage of interactions, posts and participants [exemplified in Chapter Eleven]

These last two advances also permitted the visualisation of the intricate relationship structure of the posts that allowed an additional demonstration of the presence of transformative pedagogical interactions and contingent discourse/conversations [see Claim 2 below].

Overall the whole approach derived allowed a rigorous and valuable exploration to address both the complexity and the vast amount of data studied. The claim is in effect that a

methodology including several new elements has been assembled and its use resulted in successful completion of the task, a task that had not been done before on a web-board of this size and unique nature.

Claim 2: Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy

The work of van Lier [1996], developed in a classroom situation, pre-dates the widespread use of the Internet. The theoretical frameworks encompassed within his work all play a part in making sense of and demonstrating the rich learning experience provided by AskNRICH. van Lier's theories on the categorisation of types of pedagogical interactions and his conceptualisation of multiple Zones of Proximal Development were used widely in the analysis of teaching and learning in AskNRICH in all three Perspectives.

The Exemplar Threads Perspective [Chapter Nine] shows that the effect two of the Posting Protocols on the AskNRICHers' actions/behaviours has the immediate consequence of establishing a Socratic-Style of Dialogue [see Thesis Section 7.5.1] that permeates the threads, leading to the adoption of scaffolding strategies. This Perspective also shows that the teaching and learning interactions taking place within exchanges have a spontaneity and conversational tone that leads to a high degree of contingency, a key concept within van Lier's categorisation of pedagogical interactions.

The analysis of the threads within the Three Threads Perspective [Chapter Eleven] presented a more detailed consideration of contingency levels by examination of the extent to which constituents of contingency as represented diagrammatically by van Lier [1996: 179] were present in the threads. Findings showed that the teaching and learning interactions present within AskNRICH generally would fit more towards the outer limits of each of the constituents of contingency and thus the AskNRICHers have the opportunity to engage more freely in a transformational pedagogy.

The Case Study Perspective [Chapter Ten] demonstrated that Peter's multiple teaching and learning roles were consistent with van Lier's four-part ZPD model [van Lier 1996: 194]. The opportunities afforded within AskNRICH to (i) engage in a variety of roles, and with a variety of peers who maybe more, equally or less capable and (ii) initiate and manage the conversations, provide a potentially transformational effect both in its environment and on the mathematical development of an AskNRICHer.

Thus this study has shown how van Lier's series of ideas/concepts concerning transformational pedagogical interactions can be appropriated into the virtual world, at least as represented by AskNRICH.

Claim 3: The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits

This is a two-part claim, the first of which is that AskNRICH provides a harmonious mathematical experience and this harmony produces learning benefits. The theoretical frameworks selected to underpin the analysis reported in Chapters Nine to Eleven directly relate to teaching and learning concepts. However, as the Figures in Chapter Twelve portray, these concepts provide only a partial view of the AskNRICH environment. The complete, wholistic picture can only be formed by simultaneously considering the five interdependent groupings and their interplay together. Bringing in the People Characteristics, the Structural/Medium and the Social Presence groupings leads to the inference, in Chapter Fourteen, of the 4Cs cooperation, collaboration, consideration and care being at the heart of AskNRICH [Figure 14.1 Chapter Fourteen p3/Thesis p285], producing a self-sustaining and self-perpetuating environment with each of the 4Cs resulting from and contributing to the other three. These four qualities give the sense of harmony that abides within the AskNRICH environment: a place available for anyone who wishes to engage in doing mathematics; a place for doing mathematics in a way that is analogous to 'playing the game', as professional mathematicians might be perceived as doing, rather than practising the 'dribbling' [Section 11.x Chapter Eleven p15/Thesis p54]; an inclusive, open and pleasant place of nurture in which to learn mathematics.

The second part of this claim relates to use of the Internet by AskNRICH, which would not and could not exist without it[1]. The activities within AskNRICH and the outcomes as a consequence of these activities have been shown as purposeful in supporting learning, thus demonstrating an exemplary, simple, worthwhile and beneficial use of Internet-mediated communication. Just by providing the opportunity to communicate with like-minded people, the Internet here has made a tangible and real difference [Abbott 2001: 88] for a group of young people [see quotations from AskNRICHers across Chapters Eight to Twelve]. Given the 'bad press' that much of the Internet receives, the AskNRICH narrative is, I would argue, undoubtedly an example worthy of wide dissemination.

Claim 4: AskNRICH can be successfully characterised using a concept of 'place' based on a modification of Gee's model of an Affinity Space through the introduction and definition of two new concepts, Pupil Learning Place [PLP] and Second Learning Place [SLP]

As presented in Chapter Fourteen: the voluntary and fluid nature of participation, with some participants only being in either the teaching or learning role; the need of the individual to find the solution to their (for, and at, the moment) problem, and the fact that someone will always know the answer, all ultimately provide strong arguments to suggest that what goes on within the AskNRICH virtual world is of greater importance than the individuals themselves, i.e. 'placeness' is paramount.

In defining the final characterisation of AskNRICH, this study introduces and defines two new concepts [see Chapter Fourteen] that can be added to the collection of definitions describing virtual learning worlds: Pupil Learning Place [PLP] and, as one type of PLP, Second Learning Place [SLP]. PLP is a generic term for *any* place where *learning* occurs *with the aid of teaching* amongst people who are not necessarily of *any* specific age. An SLP is characterised, in general and specific terms respectively, by the 'safe haven commune' in which to learn, and by the virtual world that is AskNRICH. AskNRICH, and by equivalence, an SLP, is where people pursue a common interest (in an academic curriculum subject) in a

[1] Incidentally, it is worth noting that AskNRICH is not reliant on any of the recent, more sophisticated innovations in the technology (Web 2.0), having used the same unpretentious interface and thrived for more than a decade.

collaborative, **c**ooperative environment showing **c**onsideration and **c**are for others with participants either aiding the pursuit or being aided in the pursuit, or doing both.

Although the concept of an SLP derives from Gee's [2004, 2005] work on Affinity Spaces there are important differences that make SLPs worthy of having a separate identity [see Thesis Chapter Fourteen p289]. AskNRICH in essence exists to provide the means for students of mathematics, predominantly at school or undergraduate level, to develop their own mathematical knowledge. That is participants of AskNRICH are undertaking independent, personal learning in a curriculum based subject at an alternative (second) location to that of formal education. The fundamental nature of an SLP springs from this definition of location, purpose and activity together with the qualities of a safe haven commune underpinned by the 4Cs as referred to above. In other words these aspects are required for the existence of an SLP and are key to producing the benefits of the harmonious learning experience referred to in Claim 3.

Claim 5: The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools

This claim results from reflection on elements that had been 'discovered' during the characterisation of AskNRICH and on the potential they might have to influence practice in school-based teaching and learning. This claim rests on the fact that the teaching and learning strategies employed by the AskNRICHers, underpinned by the Posting Protocols, have been demonstrated to be successful within the AskNRICH environment [see Chapters Nine to Twelve]. Thesis Chapter Two provides a Literature Review that portrays a generally 'poor' tone of mathematics teaching and consequent 'unhappy' learning experiences. Thus it may be argued that the successes achieved through the nature of AskNRICH as a learning place are worthy of wider consideration and possible experimentation within the school environment.

In particular, one lesson drawn is the effectiveness of the Posting Protocols (within AskNRICH as an SLP) imploring the helper not to give the answer and the learner to share current progress. As stated in Claim 2, these two Protocols encourage a conversational tone

beyond IRF and provide *natural* opportunities to become involved in types of talk that are less reliant on the teacher taking the lead. Furthermore, the elements of the environment present within an SLP (as detailed above in Claim 4) enable a 'happier' learning experience, that safe haven in which to commune [see Chapter Fourteen] providing a further quality with the potential to complement school-based learning. Finally, the learning experience in AskNRICH results from the successful strategies of peers who are of mixed age and mixed experience. In AskNRICH there is no apparent power relationship between helper and learner, with each appearing to embrace the asymmetry of knowledge and both gaining from the interactions [see for example Chapters Eight and Ten]. Creating opportunities for such 'mixed peers' to interact is again a quality whose benefits are worthy of consideration in a school context.

Having presented and discussed the claims proposed as contribution to knowledge, the remaining two parts of the chapter contain extracts from the Thesis sections on (a) limitations to the study and (b) the implications of the research.

15.4 Limitations to the Study

There are some limitations to be discussed concerning the methodological approach and the organisation of the exploration of AskNRICH. Any chosen methodology is likely to bring with it intrinsic limitations, and those associated with the qualitative methods used here are already well known and documented in the literature. In this study the impact of, for example, subjectivity in interpreting text with latent content and reliability and validity of coding by a lone researcher is considered and steps taken to minimise such methodological limitations are reported at length [see Thesis Chapter Six]. Chapter Six also reports the constraints on choosing a method appropriate to characterising AskNRICH with all its complexities and special nature together with the vast amount of data available for study. There is scope for other researchers to choose different options in determining their approach, but throughout the thesis care has been taken to report and explain the choices and decisions and resulting paths through an unknown territory. Thus I am content that the coverage of AskNRICH achieved can support the claims presented and stands up to scrutiny.

This research has limited its focus to just one mathematics web-board, AskNRICH, albeit a well-established one now in its second decade of use and unchanged since its inception. In addition to the longevity and stability of AskNRICH, its roots remain firmly placed within the mathematics education department that conceived it and thus ensures that it is a web-board worthy of serious educational research study. However, findings from the research conducted here cannot start to be transferred to a general population perspective without accounting for two potential limiting factors relating to the participants. Firstly, the number of active participants i.e. AskNRICHers at any one time remains relatively small [within the low hundreds]. However this figure does not include lurkers, who the technical team believe, account for around 90% of all AskNRICH traffic. A second, previously acknowledged, limiting factor is that AskNRICHers tend to be extremely high-attainers in mathematics – and it may be speculated probably in other subjects too, though this remains indeterminable. Nevertheless, AskNRICH was established because the founders felt that these high-attainers' special needs had been neglected. As the findings show AskNRICH has provided the opportunity for such students to be in 'virtual' touch with like-minded peers and thus it serves its purpose.

On further reflection, my declaration and confession at the very beginning of this thesis [Chapter One p1/Thesis p22] signals the clear view in my own mind of what constitutes good/effective teaching and learning of mathematics and consequently the values that I hold in relation to them. Thus the vision of what is to me good teaching and learning has clearly governed the judgements made in the analyses in this thesis. The excitement, enthusiasm for, and enjoyment of doing mathematics shown by the AskNRICHers in the threads in the three perspectives, and frequently in the many other threads considered in the process of analysis, vividly reflects my own pleasure in learning. Similarly, the AskNRICHers' pedagogy parallels my own and the associated values.

Thus it might be argued that it is hardly surprising that AskNRICH emerges from this Study as a stimulating environment and, to me, exemplifies my values. Indeed, my values and personal experience contributed to shaping the Posting Protocols. Furthermore, NRICH and AskNRICH were set up to provide a resource for (high-attaining) people who were keen to pursue their study beyond that of 'normal' school mathematics. Hence it might be

presupposed that the 'discovered' characteristics would be inherently present given the web-board design and the high-attaining nature of the participants [see Chapter Nine p25/Thesis p208]. Nonetheless, I was genuinely surprised and gratified by the tangible and pervasive effect of the Posting Protocols [see Chapter Twelve p4/Thesis p269] exemplified by the degree of self-moderation apparent on the web-board [see for example Chapter 9 p28/Thesis p211 and Chapter Ten p26/Thesis p239]; there is no built-in guarantee, beyond a light-touch moderation, that the AskNRICHers will inevitably and routinely conform. Thus, although the designers of AskNRICH had set out to create an environment based on their imaginings of the needs of hypothetical AskNRICHers, the existence of the characteristics that have been drawn out or 'discovered' is firmly rooted in the methodology and findings of the analysis of the AskNRICHers' work.

15.5 Research Implications

The work undertaken and the claims made potentially contribute to a variety of fields: firstly to use of the Internet for learning and secondly in defining communities and virtual spaces. The work also adds a new, potentially highly influential perspective to the field of Mathematics Education that emanates from AskNRICH's innovative use of a web-board technology and the implications for transformation of school-based practices.

The new techniques developed in this study were designed specifically to meet the challenges posed by the complexities, size and nature of the task of analysing the AskNRICH web-board. However, there is potential for future researchers analysing web-boards, which may or may not be restricted to those similar in nature to AskNRICH, to adopt or adapt these techniques or elements of them. Thus there is also a possible contribution to research methods associated with CMCs.

This study has demonstrated the far-reaching effects of the Posting Protocols on the nature of and behaviours within the AskNRICH environment, a lesson with potentially widespread applications in using Internet-mediated communication for peer teaching and learning. The final characterisation of AskNRICH in terms of a space rather than a community widens a debate [see Thesis Chapter Fourteen] already begun by e.g. Gee [2005] and Wubbels [2007].

The findings of this study demonstrate that the AskNRICH environment successfully supports learning amongst the AskNRICHers whose participation shapes interactions to offer a rich mathematical experience to all. The environment is neither a classroom nor constrained by any assessment or examination systems [Hatch 2002; Schoenfeld 1994; Watson 2006]. In addition the AskNRICHers show themselves to be high-attainers in the subject (if not others). Nevertheless such differences should not preclude considering placing similar strategies within the school-based environment. After all AskNRICH has been made sense of here through using theories developed for the classroom and, as the literature shows, adopting the findings from these theories has long been advocated by educationists, although implementation may not have been as full as desired [Boaler 1997; Watson 2006]. This study has provided the evidence that what takes place within AskNRICH and how it takes place leads to successful learning within an Internet environment. As Claim 5 argues the successes achieved through the nature of AskNRICH as a learning place have the potential to be applied more widely back within the school environment. Thus this could imply, for example, some adoption/adaptions of the Posting Protocols that should ensure a quantity and quality of more symmetrical talk and the creation of vertical aged groupings. Taken together these measures have the potential to begin to invoke an atmosphere resembling that of AskNRICH. Whether schools retain their current format or not and whatever impact Internet environments make on education, the lessons from this study should still apply.

Laughter and Tears were sometimes not far away as I read many of the AskNRICH posts. The *joie de vivre* shown by the AskNRICHers as they pursued their mathematics study exemplified all that I have held dear to my beliefs of what true education should mean.

Their Life and their Work is a Tale Worth Telling

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